Non-linear dynamics in ecology
Incl epidemiology

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Nonlinearity in Ecology:
A Short Introduction

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Nonlinearity in Ecology:
A Short (non mathematical) Introduction

- Interest of Chaotic Dynamics in Ecology and Population Biology
- What is a chaotic dynamics?
- The Project Tribolium
- Chaos Always a Topic of Interest in Ecology
- Examples of Chaotic Dynamics in Ecological Models
A Short Introduction

Interest of Chaotic Dynamics in Ecology and Population Biology

- Understanding the complex population oscillations observed in Nature is one of the main objective of Population Biology
- Frequently, the observed dynamics are complex
Examples of complex dynamics in Ecology

A Short Introduction

Interest of Chaotic Dynamics in Ecology and Population Biology

Some of the *Lucilia* time series, two of the Control and two of the Cadmium-treated populations. The top four panels are the time series of pupal counts.
A Short Introduction

Interest of Chaotic Dynamics in Ecology and Population Biology

Examples of complex dynamics in Ecology

Figure 1. The time series of pooled small rodent abundance at Kilpisjärvi, Finnish Lapland. The graph shows two data points per year, representing the spring (open symbol) and autumn densities (closed symbol), respectively. Change during the summer is shown by a continuous line, change during the winter by a broken line.

Time series of small rodent abundance at Pähkäläjarvi, Finnish Lapland. Results are shown for open (lower panel) and forested (upper panel) habitats, and in each case the results are shown separately for the numerically dominant species and the pooled data (the dominant species are C. glareolus in forests and M. agrestis in open habitats).
A Short Introduction

Interest of Chaotic Dynamics in Ecology and Population Biology

Examples of complex dynamics in Ecology

Pacific Sardine
between 1959 and 1996

Peruvian Anchovy
between 1920 and 1939

Diatoms
between 1920 and 1939
Understanding the complex population oscillations observed in Nature is one of the main objectives of Population Biology.

Frequently, the observed dynamics are complex.

The dominant paradigm was:

- Simple patterns \(\rightarrow\) simple causes
- Complex patterns \(\rightarrow\) complex causes

1974-1976 Bob May demonstrated that simple mechanisms can generate very complex dynamics.
A Short Introduction

Interest of Chaotic Dynamics in Ecology and Population Biology

Biological Populations with Nonoverlapping Generations:
Stable Points, Stable Cycles, and Chaos

Abstract. Some of the simplest nonlinear difference equations describing the growth of biological populations with nonoverlapping generations can exhibit a remarkable spectrum of dynamical behavior, from stable equilibrium points, to stable cyclic oscillations between 2 population points, to stable cycles with 4, 8, 16, . . . . points, through to a chaotic regime in which (depending on the initial population value) cycles of any period, or even totally aperiodic but bounded population fluctuations, can occur. This rich dynamical structure is overlooked in conventional linearized analyses; its existence in such fully deterministic nonlinear difference equations is a fact of considerable mathematical and ecological interest.

Nature Vol. 261 June 10 1976

review article

Simple mathematical models with very complicated dynamics
Robert M. May*

First-order difference equations arise in many contexts in the biological, economic and social sciences. Such equations, even though simple and deterministic, can exhibit a surprising array of dynamical behaviour, from stable points, to a bifurcating hierarchy of stable cycles, to apparently random fluctuations. There are consequently many fascinating problems, some concerned with delicate mathematical aspects of the fine structure of the trajectories, and some concerned with the practical implications and applications. This is an interpretive review of them.
At the end of the 70’s the “chaos revolution” (determinist chaos) invades numerous scientific domains but

- Henri Poincaré (1905) : « une cause très petite qui nous échappe détermine un effet considérable que nous ne pouvons pas voir, et alors nous disons que cet effet est du au hasard »

- Chaos ~ ordered disorder generated by simple deterministic processes

- Chaos ~ apparent random dynamics generated by simple deterministic processes
A Short Introduction

What is a Chaotic Dynamics?

- Apparent random dynamics
Random dynamics generate by simple deterministic processes.
What is a Chaotic Dynamics?

- High sensitivity to initial conditions
But the dynamics is confined in the phase space describing a "strange attractor" with fractal structure.
What is a Chaotic Dynamics?

- The dynamics is confined in the "strange attractor".
- Macroscopic predictability but microscopic unpredictability.
Interest of apparent random dynamics generated by simple deterministic mechanisms

As ecological systems are complex it was likely that deterministic chaos may be observed in these systems

Reconciliation of the 2 main paradigms used in population dynamics
  - A purely deterministic approach
  - A purely stochastic approach

But Chaos ~ non-equilibrium: This is in discordance with a classical concept in Ecology: the climax notion
A Short Introduction

Interest of Chaotic Dynamics in Ecology and Population Biology
A Short Introduction

The Project Tribolium
Cannibalism of the unmoving stages by the mobile stages

A Short Introduction

The Project Tribolium

Cannibalism of the unmoving stages by the mobile stages

==> Non Linearity
A Short Introduction

The Project Tribolium

\[ L_t = b . A_{t-1} . \exp\{-c_{EL}.L_{t-1} - c_{EA}.A_{t-1}\} \]

\[ P_t = L_{t-1}.(1 - \mu_L) \]

\[ A_t = P_{t-1}.(1 - \mu_P).\exp\{-c_{PA}.A_{t-1}\} + A_{t-1}.(1 - \mu_A) \]

- \( L \): larva ; \( P \): pupa ; \( A \): adult
- \( \mu_A, \mu_P \): mortality rates of \( P \) et \( A \)
- \( C_{EL}, C_{EA}, C_{PA} \): cannibalism rates
A Short Introduction

The Project Tribolium

- Analysis of the LPA model (modifying the parameters: $\mu_A$ or $C_{PA}$)
- Experimentation (by adding or removing some adults or pupas)
A Short Introduction

The Project Tribolium

$\mu_A = 0.96$

The upper graph is a parameter plane map showing the boundaries defined by different types of attractors of the LPA model for differing values of $\mu_A$ and $c_{pa}$. 
A Short Introduction

The Project Tribolium
A Short Introduction

The Project Tribolium
Chaos always a domain of interest in Ecology

Preventing Extinction and Outbreaks in Chaotic Populations

Disease in prey population and body size of intermediate predator reduce the prevalence of chaos—conclusion drawn from Hastings–Powell model

Probing chaos and biodiversity in a simple competition model
Interest in Ecology: even in top journals!

LETTERS

Reduced mixing generates oscillations and chaos in the oceanic deep chlorophyll maximum

Jef Huisman1*, Nga N. Pham Thi2*, David M. Karl3 & Ben Sommeijer2

LETTERS

Chaos in a long-term experiment with a plankton community

Elisa Benincà1,2*, Jef Huisman1*, Reinhard Heerkloss3, Klaus D. Jöhnk1†, Pedro Branco1, Egbert H. Van Nes2, Marten Scheffer2 & Stephen P. Ellner4
Examples of chaotic dynamics in ecological models

Trophic network models

\[
\frac{du_i}{dt} = a.u_i.(1 - u_i/k_o) - \alpha_1.f_1(u_i, v_i)
\]
\[
\frac{dv_i}{dt} = -b_i.v_i + \alpha_1.f_1(u_i, v_i) - \alpha_2.f_2(u_i, v_i)
\]
\[
\frac{dw_i}{dt} = -c.(w_i - w^*) + \alpha_2.f_2(u_i, v_i)
\]

\[f_j(x, y) = \frac{x.y}{1 + k_j.x}\]
Examples of chaotic dynamics in ecological models

Trophic network models

\[ \frac{du_i}{dt} = a_iu_i(1 - u_i/k_i) - \alpha_1f_1(u_i,v_i) \]

\[ \frac{dv_i}{dt} = -b_i v_i + \alpha_1f_1(u_i,v_i) - \alpha_2f_2(u_i,v_i) \]

\[ \frac{dw_i}{dt} = -c_i(w_i - w^*) + \alpha_2f_2(u_i,v_i) \]
Forced SEIR model in epidemiology

Examples of chaotic dynamics in ecological models

\[
\frac{dS}{dt} = \mu N - \beta(t) \frac{SI}{N} - \mu S
\]

\[
\frac{dE}{dt} = \beta(t) \frac{SI}{N} - (\sigma - \mu) E
\]

\[
\frac{dI}{dt} = \sigma E - (\gamma - \mu) I
\]

\[
\beta(t) = \beta_0 \left( 1 + \beta_1 \cos(\omega t) \right)
\]

Constructing a bifurcation diagram. The top 3 panels depict time-series data for the \(SEIR\) model with different levels of seasonality (\(\beta_1 = 0.025, 0.05\), and \(0.25\), respectively). The arrows at the top of the panels indicate the points when the time series are sampled in order to construct the bifurcation diagram below. The parameters used to generate these panels were \(\mu = 0.02\) per year, \(\beta_0 = 1250\), \(1/\sigma = 8\) days and \(1/\gamma = 5\) days. All simulations were started with \(S(0) = 6 \times 10^{-2}\) and \(E(0) = I(0) = 10^{-3}\).
Nonlinearity at Work in Populations:
From Theory to Observations and Experiment(s)

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Examples of Nonlinearity at Work

- **Intermittent Rarity**
  (Mainly with Régis Ferrière)

- **Spatial Population Synchrony**
  (In collaboration with Lewi Stone)

- **Effect of Noise on Population Synchrony**
  (In collaboration with Lewi Stone)

- **Seasonal Flu Epidemics**
  (Mainly done by Sébastien Ballesteros)
Examples of Nonlinearity at Work

- Intermittent Rarity
- Spatial Population Synchrony
- Noise Induced Population Synchrony
- Seasonal Flu Epidemics
Interruption Rarity

Epidemics

Monthly reported cases of Dengue in Pacific Islands between 1970 and 1999
Intermittent Rarity

Marine Communities

Pacific Sardine

Peruvian Anchovy
between 1959 and 1996

Diatoms
between 1920 and 1939
The transversal Lyapunov exponent $\lambda$ or the invasion exponent characterizes the transversal stability of “$Y = 0$” in an invariant sub-space.

Intermittent Rarity

Competition between $X$ a resident species and $Y$ a rare species.
Interruption Rarity

Competition between
\( X \) a resident species and
\( Y \) a rare species

\[ \lambda_\perp < 0 \]

"\( Y = 0 \)" is stable
\( Y \) the rare species cannot invade the community

\( Y = 0 \)
invariant sub-space
\( \lambda_\perp > 0 \)

“\( Y = 0 \)” is unstable

\( Y \) the rare species can invade the community

\( Y = 0 \)

invariant sub-space

**Intermittent Rarity**

Competition between

\( X \) a resident species and

\( Y \) a rare species
But $\lambda_\perp(t)$ fluctuates and the distribution of $\lambda_\perp$ can overlap 0

Intermittent Rarity

Competition between $X$ a resident species and $Y$ a rare species

- If the mean of $\lambda_\perp(t)$ is slightly negative => “Riddled Basins”
- If the mean of $\lambda_\perp(t)$ is slightly positive => “On-Off” Intermittency
“Y=0” is unstable in the transverse direction and in average the trajectories are repelled from Y=0 (*On State*)

**BUT** there are some trajectories that are attracted by Y=0 (*Off State*)
The distribution of rarity phases follows a power law:

$$P(D) = D^\beta$$

with: $$\beta = -\frac{3}{2}$$
Competition between populations

Ricker-Gatto Model

\[ X_{t+1} = X_t \left( r_1 \exp(-X_t - Y_t) + s_1 \right) \]

\[ Y_{t+1} = Y_t \left( r_2 \exp(-X_t - Y_t) + s_2 \right) \]
Intermittent Rarity

Fluctuating Environment

Ricker Model

\[ Y_{t+1} = Y_t \exp((r_2 - V_t) - Y_t) \]

\[ V_t \sim N(r_2 - \epsilon, \sigma^2) \]
Prey-Predator or Host-Parasitoid Interactions

Model

Host –
Specialist Parasitoid (Y) –
Generalist Parasitoid (Z)

\[ X_{r+1} = X_r \exp \left( 1 - \left( 1 - \frac{X_r}{k_1} \right) \right) \cdot \left[ 1 - \frac{a_2 \cdot Y_r}{k_2} \right]^{k_2} \cdot \exp(-a_3 \cdot Z_r) \]

\[ Y_{r+1} = X_r \left( 1 - \left[ 1 - \frac{a_2 \cdot Y_r}{k_2} \right]^{k_2} \right) \cdot \exp(-a_3 \cdot Z_r) \]

\[ Z_{r+1} = c_3 (X_r + Y_r) \left( 1 - \exp(-a_3 \cdot Z_r) \right) \]
Intermittent Rarity

Marine Communities

Pacific Sardine
between 1959 and 1996

Peruvian Anchovy
between 1920 and 1939

Diatoms
Intermittent Rarity
Marine Communities

\[ \beta = -\frac{3}{2} \]

Pacific Sardine
between 1959 and 1996

Peruvian Anchovy
between 1920 and 1939

Diatoms
between 1920 and 1939
Intermittent Rarity
Marine Communities
“On-Off” intermittency is a potential explanation to intermittent rarity.

Intermittent rarity can arise due to weak invading capacities of a species in fluctuating communities and in fluctuating environment: intermittent rarity develops on the edge of competition exclusion.

In such cases, the distribution of the duration of rarity phases follows a strong regularity.
Examples of Nonlinearity at Work

- Intermittent Rarity
- Spatial Population Synchrony
- Noise Induced Population Synchrony
- Seasonal Flu Epidemics
Spatial Population Synchrony

- Spatial synchrony is a general phenomenon common to many taxa

![Graph showing population synchrony](image)

- Lynx
- Lepidoptera
Spatial Population Synchrony

- Spatial synchrony is a general phenomenon common to many taxa
- Different reasons
  - Dispersal or Migration
  - Common Nomadic Predators
  - Common Environmental Forcing
    - “The Moran Effect”
2 populations regulated by the same linear density dependence process become synchronized when exposed to common environmental fluctuations \( \rho_{x,y} \approx \rho_{\epsilon_x,\epsilon_y} \)

The Moran effect is a linear theory
Numerous works demonstrated that non linearity undermines the synchronization. The Moran effect alone is unable to synchronize complex identical dynamics.

e.g. Bjørnstad et al 1999
A tri-trophic model with 2 non connected patches ($i=1,2$):

\[
\frac{du_i}{dt} = a_i u_i (1 - u_i / k_o) - \alpha_1 f_1(u_i, v_i) + \xi_{ui}
\]

\[
\frac{dv_i}{dt} = -b_i v_i + \alpha_1 f_1(u_i, v_i) - \alpha_2 f_2(u_i, v_i) + \xi_{vi}
\]

\[
\frac{dw_i}{dt} = -c_i (w_i - w^*) + \alpha_2 f_2(u_i, v_i) + \xi_{wi}
\]

with

\[
f_j(x, y) = \frac{x \cdot y}{1 + k_j \cdot x}
\]

\[
b_1 \neq b_2
\]

\[
\xi_{ji} = A \cdot \cos(\Omega(t) \cdot t) + \xi_{ji}
\]
This imperfect synchrony is attained with ‘forcings’ (A) that have always very weak influences on the dynamical features

\[ \xi_{ji} = A \cdot \cos(\Omega(t) \cdot t) + \zeta_{ji} \]

A  \( \xi_{ji} = 0 \)
B  \( \xi_{ji} = \xi_{ji} \)
C  \( \xi_{ji} = A \cdot \cos(\Omega(t) \cdot t) \)
D  \( \xi_{ji} = A \cdot \cos(\Omega(t) \cdot t) + \zeta_{ji} \)
E  \( \xi_{ji} = A \cdot \cos(\Omega(t) \cdot t) \)
Generalization of the Moran Effect

The UPCA Model - Results
Generalization of the Moran Effect

\[ \phi(t) = 2\pi \left( \frac{t-t_n}{t_{n+1}-t_n} + n-1 \right) \]

\[ \psi(t) = \phi(t) \mod 2\pi \]

Synchronization

if: \[ \Delta \varphi = \text{constant} \]
Distribution of the Cyclic Phase Differences
\[ \Delta \phi = [\phi_1(t) - \phi_2(t)] \]

Generalization of the Moran Effect
For different noise features

Generalization of the Moran Effect

The UPCA Model - Results
For different noise features and N patches with different $b_j$ (N=10)
For different noise features and $N$ patches with different $b_i$ ($N=30$)

$$\xi_{jk}^{i+1} = A \cos[\Omega(t)(t - i\delta)] + \xi_{jk}^i$$

The UPCA Model - Results
Generalization of the Moran Effect

The UPCA Model - Conclusions

- Generalization of the **Moran Effect** for complex dynamics
- A common noisy quasi-periodic forcing can bring into synchrony the phases of population abundance and modulates identically the dynamics of N distant and non-linked populations
Corroboration of these Theoretical Results

- Observed Time Series of Population Synchrony
- Experimentation on Rotifer Populations
\[ \phi(t) = 2 \pi \left( \frac{t-t_n}{t_{n+1}-t_n} + n - 1 \right) \]
\[ \psi(t) = \phi(t) \mod 2\pi \]

Synchronization or relation between the 2 series if:
\[ \Delta \varphi = \text{constant} \]
Distribution of the Cyclic Phase Differences

\[ \Delta \phi = [\phi_1(t) - \phi_2(t)] \]

Normalized Entropy \( Q \) is employed to quantify these distributions
Observed Population Synchrony

Sheep Population in the St Kilda Archipelago

Grenfell et al 1998
Observed Population Synchrony

Grey-Sided Vole Population in northern Finnish Lapland

Hansen et al 1999
Rotifer Experimentation

Rotifer populations (Brachionus) influenced by fluctuations in prey density (Chlorella)

- Populations started out of phase
- 1 control + 2 treatments
  - control C
  - prey oscillations T1
  - prey oscillations T2
- 4 replicates for each treatment

Rotifer populations initially antiphase synchronized are resynchronized by common prey density fluctuations
Rotifer Experimentation

Rotifer populations (Brachionus) influenced by fluctuations in prey density (Chlorella)

Rotifer populations initially antiphase synchronized are resynchronized by common prey density fluctuations.
Spatial Population Synchrony and the Moran Effect

Conclusions

- Even weak environmental fluctuations are capable to bring into synchrony the phases of populations
- Environmental oscillations can act as a “pacemaker” that modulates complex dynamics, even if it modify slightly the dynamical features of the population
- Phase analysis ≈ a non-linear tool for the analyses of weak interactions in irregular, non-stationary and noisy time series that are common in Biology
Examples of Nonlinearity at Work

- Intermittent Rarity
- Spatial Population Synchrony
- Noise Induced Population Synchrony
- Seasonal Flu Epidemics
Noise Induced Synchronisation

Generalization of the Moran Effect

Hastings & Powell, 1991

\[
\begin{align*}
\frac{dx_i}{dt} &= r \cdot x_i(1 - K \cdot x_i) - f_1(x_i) \cdot y_i + D \cdot \xi_{ui} \\
\frac{dy_i}{dt} &= -d_{vi} \cdot y_i + f_1(x_i) \cdot y_i - f_2(y_i) \cdot z_i + D \cdot \xi_{vi} \\
\frac{dz_i}{dt} &= -d_{wi} \cdot z_i + f_2(y_i) \cdot z_i + D \cdot \xi_{wi}
\end{align*}
\]

with

\[f_k(u) = \frac{a_k \cdot u}{1 + b_k \cdot u}\]

\[\xi_{ji} \text{ a gaussian component}\]

\[D \text{ noise intensity}\]
Noise Induced Synchronisation

Generalization of the Moran Effect

Hastings & Powell, 1991
Noise Induced Synchronisation

Generalization of the Moran Effect

Hastings & Powell, 1991
Noise Induced Synchronisation

Generalization of the Moran Effect

- Generalization of the Moran Effect for complex dynamics
- A common noisy component can synchronize the complex dynamics of population abundance
- There is some optimal noise intensity that can tune and maximize some ordering behavior of a populational systems (Exogenous Stochastic Coherence)
Examples of Nonlinearity at Work

- Intermittent Rarity
- Spatial Population Synchrony
- Noise Induced Population Synchrony
- Seasonal Flu Epidemics
Influenza epidemics in temporal areas appear each year but their amplitude are highly variable.
Chaotic but Regular Epidemics

UPCA dynamics is a potential explanation

The periodicity of pneumonia and influenza mortality and excess mortality rates. Monthly pneumonia and influenza (P&I) death rates and excess death rates (above the baseline mortality due to other respiratory pathogens) in the United States from 1959 to 2001 are shown.

(a) Weekly cases per 10,000 in the Netherlands and (b) weekly cases per 100,000 of influenza in France: the main types or strains for each year are labelled.
UPCA Dynamics in Flu Epidemics

A simple SIRS model that incorporate:

- External reintroduction ($\eta$)
- Seasonal forcing ($e$)
- Gradual antigenic drift ($g$)
UPCA Dynamics in Flu Epidemics

A simple SIRS model:

\[
\begin{align*}
\frac{dS}{dt} &= -\beta_0(1 + e\cos(2\pi t)) \frac{S}{N}(I + \eta) + gR \\
\frac{dI}{dt} &= -\beta_0(1 + e\cos(2\pi t)) \frac{S}{N}(I + \eta) - \nu I \\
\frac{dR}{dt} &= \nu I - gR
\end{align*}
\]
UPCA Dynamics in Flu Epidemics

Model Simulations

A

B

C
UPCA Dynamics in Flu Epidemics

Bifurcation based on the 1st Lyapunov exponent

Theor. parameters

Emp. parameters

\[ R_0 = 5 \quad \frac{1}{\nu} = 8 \text{ days} \quad \eta = 10^{-6} \quad R_0 = 2.6 \quad \frac{1}{\nu} = 2.8 \text{ days} \quad \eta = 10^{-7} \]
UPCA Dynamics in Flu Epidemics

Bifurcation based on the 1\textsuperscript{st} Lyapunov exponent

Theor. parameters

Emp. parameters

1 strain model

2 strains model
UPCA Dynamics in Flu Epidemics

Comparison with Observations

1 strain model

Israel data

Paris (Ile-de-France) data

2 strain model
UPCA Dynamics in Flu Epidemics

Robustness of these Results

- Increasing realism of the model (e.g. demography, Erlang distributions or temporary period of full cross protection)
- Stochastic version of this SIRS model
- Metapopulation with immigration and heterogeneous contacts due to age structure
UPCA Dynamics in Flu Epidemics

Robustness of these Results

- Metapopulation with immigration and heterogeneous contacts due to age structure
UPCA Dynamics in Flu Epidemics

Robustness of these Results

- Metapopulation with immigration and heterogeneous contacts due to age structure
UPCA Dynamics in Flu Epidemics

Robustness of these Results

- Metapopulation with immigration and heterogeneous contacts due to age structure
Conclusions

- Non-linear dynamics alone can induce irregularity of the epidemic amplitudes.
- This nonlinear intrinsic view with gradual evolution constitutes a minimal explanation for the high irregularity of the recurrent seasonal flu epidemics.
- These UPCA dynamics are robust to different perturbations highlighting the interest of this minimal flu model.
Nonlinearity at work in Populations

Concluding Remarks

- Concepts from nonlinear theory can help us in analyzing and understanding the complexity observed in Nature.

- For some nonlinear systems, noise must have a constructive influence (i.e. stochastic resonance or stochastic coherence).
First, nonlinearity is pervasive; second, inclusion of stochasticity is essential; and third, issues of scale are common to all quantitative approaches.
Non-stationary influence of climatic oscillations on epidemics

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Non-stationary influence of climatic oscillations on epidemics

- Introduction: Associations between Epidemics and Climate are Non-Stationary
- Description and Quantification of Non-Stationary Relationships
  - Wavelet Analysis
  - Epidemiological Examples
- Conclusion
Non-Stationary for an Epidemics

An example of measles epidemics in York (UK)

Characteristics evolve with time => Non-Stationarity
Non-Stationary Associations

Links between climatic oscillations and some quasi-periodic epidemics like Cholera in Bangladesh

Rodo et al 2002
Non-Stationary Associations

Links between climatic oscillations and some quasi-periodic epidemics like Cholera in Bangladesh

SSA

Rodo et al 2002
A clear relationship between Cholera and El Nino for:
1893-1920 and 1980-2001!
Non-Stationary Associations

- Epidemiological time series are typically noisy, irregular and **strongly non-stationary**.
- These characteristics may make it inappropriate to use the traditional methods for their analyses.
- This is also true for the analyses of mutual dependencies between time series.
Non-stationary influence of climatic oscillations on epidemics

- Introduction: Association between Epidemics and Climate are Non-Stationary
- Quantification of Non-Stationary Relationships
  - Wavelet Analysis
  - Epidemiological Examples
- Conclusion
Wavelet Analysis

- Wavelet analysis estimates the spectral characteristics of a time series as a function of time.
- Wavelet analysis decomposes a signal into time-space and frequency-space simultaneously.
- Seldom applications in Ecology and Epidemiology (e.g., Grenfell et al, 2001, about measles in UK).
The wavelet function is put in different position determined by the translation parameter, \( b \), and with different dilatation parameter, \( a \).

\[
\Psi(t) = (1 - t^2) e^{-t^2/2}
\]

\[
\Psi(t) = \pi^{-1/4} e^{i2\pi f_0 t} e^{-t^2/2}
\]

\[
\Psi(a, b) = \Psi\left(\frac{t-b}{a}\right)
\]
Wavelet Transform is defined as:

$$T(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \Psi^* (\frac{t-b}{a}) dt$$
Wavelet Analysis

Wavelet Transform:  
\[ T(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \Psi^*(\frac{t-b}{a}) \, dt \]

(and Inverse Wavelet Transform)

Wavelet Amplitude, Imaginary Part and Wavelet Modulus:

\[ \text{Re}(T(a, b)), \quad \text{Im}(T(a, b)), \quad |T(a, b)| \]

Wavelet Power:

\[ W_x(a, b) = |T(a, b)|^2 \]

Averaged Wavelet Power:  
(by analogy with the Fourier Power)

\[ P_w(f) = \frac{1}{\tau f_c C_g} \int_{0}^{\tau} |T(a, b)|^2 \, db \]

CrossWavelet Power:

\[ W_{xy}(a, b) = \frac{W_x(a, b) W_y(a, b)^*}{\sigma_x \sigma_y} \]

Wavelet Coherency:

\[ CW_{xy} = \frac{\left| \langle W_{xy}(a, b) \rangle \right|^2}{\left| \langle W_x(a, b) \rangle \right| \left| \langle W_y(a, b) \rangle \right|} \]
Quantification of Non-Stationarity

Wavelet Analysis
Quantification of Non-Stationarity

Wavelet Analysis

\[
\begin{align*}
\psi(t) &= \sin\left(\frac{2\pi t}{T_1}\right) + \sin\left(\frac{2\pi t}{T_2}\right) + \epsilon \\
\psi(t) &= \sin\left(\frac{2\pi t}{T_1}\right) + \epsilon \quad \text{for } t < ts \\
\psi(t) &= \sin\left(\frac{2\pi t}{T_2}\right) + \epsilon \quad \text{for } t \geq ts
\end{align*}
\]

2 sinusoidal signals with identical Power Spectrum

\(T_1 = 0.25\)
\(T_2 = 1\)
\(\epsilon \approx N(\mu, \sigma)\)
Quantification of Non-Stationarity

Wavelet Analysis

\[ y(t) = \sin\left(\frac{2\pi t}{T_1}\right) + \sin\left(\frac{2\pi t}{T_2}\right) + \varepsilon \]

for \( t < t_s \)

\[ y(t) = \sin\left(\frac{2\pi t}{T_1}\right) + \varepsilon \]

for \( t \geq t_s \)

\( T_1 = 0.25 \)

\( T_2 = 1 \)

\( \varepsilon \approx N(\mu, \sigma) \)
Quantification of Non-Stationarity

Wavelet Analysis

A “simple” non-stationary sinusoidal signal

\[ y(t) = 2 \sin \left( \frac{2 \pi t}{T_2} \right) \quad \text{for} \quad t < t_s \]

\[ y(t) = \sin \left( \frac{2 \pi t}{T_1} \right) \quad \text{for} \quad t_s \leq t < t_r \]

\[ y(t) = A \sin \left( \frac{2 \pi t}{T} \right) \quad \text{for} \quad t \geq t_r \]

\[ T_1 = 0.25 \quad \text{and} \quad T = T_1 + (T_2 - T_1) \cdot \frac{t - t_s}{t_r - t_s} \]

\[ A = 1 + (2 - 1) \cdot \frac{t - t_s}{t_r - t_s} \]
Quantification of Non-Stationarity

Wavelet Analysis

A “simple” non-stationary sinusoidal signal
Dengue is a peri urban disease in the tropics and subtropics, transmitted principally by *Aedes aegypti*. The biology of the vector is influenced by the climate (rainfall and temperature) => *Quid* about dengue epidemics?

A recent Nature paper by the group of Pr. Don Burke has documented traveling waves of Dengue across Thailand.

But the authors claim that these epidemic waves are not linked with climatic variables.
Quantification of Non-Stationarity

Dengue in Thailand

Travelling waves in the occurrence of dengue haemorrhagic fever in Thailand

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Dengue fever is a mosquito-borne virus that infects 50–100 million people each year¹. Of these infections, 200,000–500,000 occur as the severe, life-threatening form of the disease, dengue haemorrhagic fever (DHF)². Large, unanticipated epidemics of DHF often overwhelm health systems³. An understanding of the spatial–temporal pattern of DHF incidence would aid the

But any links with climate!
Quantification of Non-Stationarity

Dengue in Thailand

- Monthly DHF cases reported in the 72 provinces of Thailand
- Focus on the incidence in Bangkok and the averaged incidence for the rest of Thailand
Quantification of Non-Stationarity

Dengue in Thailand

Link with Nino3
Quantification of Non-Stationarity

Dengue in Thailand

Relation between Bangkok and the remaining Thailand

Importance of non-stationarity
Previous results have been extended to a new dataset with the MEI and rainfalls.
Quantification of Non-Stationarity
Quantification of Non-Stationarity
Malaria is a major public health burden around the tropics and resurgence of malaria has been documented in several African highlands.

Observed interannual cycles have been attributed to intrinsic mechanisms (Hay et al. 2002).

But it’s also known that temperature and rainfall play an important role in the interannual variability (Zhou et al. 2004; Pascual et al. 2006).

Here we have questioned the interplay between intrinsic and extrinsic mechanisms.
Weekly confirmed malaria cases reported from 1970 to 2003 in a tea plantation in Western Kenya (Kaisugu Kericho)
Quantification of Non-Stationarity

Malaria in Highlands in Kenya

[Graph showing time series data and spectral analysis]
Quantification of Non-Stationarity

Malaria in Highlands in Kenya

Discrete SIR model
Koelle & Pascual 2004
Koelle et al 2005

\[ S_{t+1} = S_t - I_{t+1} + B - D \frac{S_t}{N_t} \]
\[ I_{t+1} = \beta_t \cdot \beta_{seas} \left( \sum_{k=1:9} I_{t-k} \frac{S_t}{N_t} \right) \cdot \epsilon_t \]

non-parametric regression

\[ \Rightarrow \beta_t = f(t) \]
\[ \epsilon_t \]
Quantification of Non-Stationarity

Malaria in Highlands in Kenya

Discrete SIR model

\[ \beta_t = f(t) \]

\[ \mathcal{E}_t \]
Quantification of Non-Stationarity

Malaria in Highlands in Kenya

Link with rainfall
Quantification of Non-Stationarity

Malaria in Highlands in Kenya

Link with rainfall
Quantification of Non-Stationarity

Malaria in Highlands in Kenya

![Graph showing cases and period over time](image)
Non-stationary influence of climatic oscillations on epidemics

- Introduction: Association between Epidemics and Climate are Non-Stationary
- Description and Quantification of Non-Stationary Relationships
  - Wavelet Analysis
  - Epidemiological Example
- Conclusions
Conclusions

- Transient characteristics are very common in epidemiological time series.
- Identifying non-stationary pattern of mutual dependencies for some diseases, is just a first step.
- The next step will be to model these patterns of association. I suggest to employ state-space models coupled to bayesian techniques (Kalman filters, particle filters).
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