Numerics for Neutral State-Dependent-Delay Differential Equations.

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Abstract Let us consider the following Neutral Delay Differential Equation with State-Dependent Delay

$$\begin{cases} y'(t) = f\Big(t, y(t), y(t-\tau(t,y(t))), y'(t-\sigma(t,y(t)))\Big), \quad t \geq t_0, \\ y(t) = \phi(t), \quad t \leq t_0, \end{cases} \tag{1}$$

In general, the analysis and the numerical integration of delay differential equations such as (1) is a very difficult task, and this is essentially due to the simultaneous presence of the delayed derivative $y'(t-\sigma)$ in the function f and of the state $y(t)$ in the delay σ itself.

Moreover, for such equations, the existence and uniqueness of the solution is guaranteed for any sufficiently smooth data f, τ, σ and ϕ provided that the splicing condition

$$\dot{\phi}(t_0^-) \quad = \quad f\Big(t_0, \phi(t_0), \phi\left(\tau(t_0, \phi(t_0))\right), \dot{\phi}\left(\sigma(t_0, \phi(t_0))\right)\Big)$$

is fulfilled at $t = t_0$. If this is not the case, the derivative of the solution has a jump discontinuity at $t = t_0$ and the effect of such irregularity is twofold. On one hand, in general, such discontinuity propagates along the integration interval to subsequent points, called breaking points, where the derivative of the solution remains discontinuous. On the other hand, at any such points

the solution may cease to exist or bifurcate. In particular, the termination of the solution during the simulation of phenomena modeled by (1) is a severe obstacle in the use of the model itself which is due to the lack of the splicing condition in the choice of the initial data rather than to the weakness of the modeling equation.

The talk is devoted to the presentation of (1) as a non neutral delay differential equations with discontinuous right hand side which admits a weak solution in the sense of Filippov and Utkin. Moreover, two possible regularizations of the equation are proposed based on singular perturbations of different types depending on a small parameter ϵ. The solutions obtained are compared with Filippov's and Utkin's weak solutions and numerical experiments are shown as well.

Bibliography:

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